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Self-confirming Inflation Persistence *

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ABSTRACT

In this paper we simulate a central bank subject to the misperception that prices are indexed to past inflation in periods when firms are unable to re-optimise. It thinks, in other words, that inflation is intrinsically persistent. The central bank sets monetary policy optimally subject to this belief. The central bank updates its beliefs about indexation using a constant gain learning scheme. The data generated by such policy lead to beliefs about inflation persistence being effectively self-confirming in a wide variety of setttings. These results offer a tentative answer to why it appears that inflation is persistent at some times and in some countries, and at others not. The answer is that policymakers sometimes believe inflation to be persistent, and sometimes do not.

* This paper is preliminary and incomplete - please do not quote without the authors' permission. We thank, without prejudicing, for helpful conversations, Martin Ellison, Richard Harrison, Jan Vlieghe. The views expressed in this paper are those of the authors and not necessarily those of the Bank of England nor the Bank's Monetary Policy Committee.

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1 Introduction

In this paper we demonstrate how a central bank's misperception that there is intrinsic inflation persistence can be self-confirming. Our results sketch an explanation for why inflation persistence appears in the data in some countries and for some monetary regimes, but not in others, and why monetary policy is correspondingly accommodative of shocks to inflation.

The story we tell is of a central bank who begins with the notion that there is indexation in price-setting: that when firms are constrained from reoptimising prices, they uprate their prices by some linear function of past observed inflation. Our central bank is engaged in a process of perpetual learning about the value of the coefficient on past inflation in this linear function. Each period, monetary policy is set optimally, conditional on this belief, with the qualification that the uncertainty in its estimates, and the impact of monetary policy on the future value and precision of these estimates is ignored. We simulate a central bank behaving like this in three realities, none of which feature indexation. In one, prices are set a la Calvo (1983). In another, prices are flexible. In a third, prices are set according to a model devised by Juillard *et al* (2006). In that model, just as in Calvo (1983), firms receive a signal that they can re-optimise prices. But in addition, firms also get to choose the linear indexation function that they get to use in the future. In all three realities, consumers and firms are rational and take expectations over the central bank's learning process. We solve the resulting non-linear rational expectations system using a parameterised expectations algorithm. Each model is peturbed by a shock to the desired mark-up of firms - which monetary policy seeks to respond to in what it thinks is the appropriate way. We experiment with white noise and persistent processes for this disturbance, where in each case we ensure that the policymaker knows with certainty the law of motion for this shock.

In many of our settings, the belief that inflation is persistent is effectively self-confirming. In some cases, there is a tendency for central bank beliefs about the indexation parameter to fall, but this is usually very slow, and even (under flexible prices) infinitessimally so. With persistence in our mark-up process, we can generate fluctuations in beliefs about inflation persistence.

Our work is most immediately inspired by Cho *et al* (2002). They show how a high inflation equilbrium is the outcome of a process in which a policymaker's misconception that there is a long run Philips Curve (which suggests it is worth buying lower unemployment with higher inflation) is self-confirming. Ellison and Yates (2007) introduced a motive for stabilisation into that model and showed how the high inflation equilibrium was also one in which changed the higher moments of inflation, not just its mean: in their setting, the high inflation state was also one in which it was more volatile. In our paper, we set up a similar thought experiment to see if a misperception about a higher moment of inflation, this time inflation persistence, can be self-confirming. Our answer is a qualified yes.

Our focus on inflation persistence derives from the widespread interest documenting and accounting for it. One aspect of this is the work establishing that monetary policy shocks have protracted effects on real variables and on inflation. (See, for example, Christiano *et al* (1999)). This was noted to be inconsistent with the sticky price models of Calvo (1983) and Rotemberg (1983). Non-microfounded models like Buiter and Jewitt (1981) and Fuhrer and Moore (1995) were advanced as being more data-congruent. Later, the indexation model was put forward as a way to modify the sticky price model. This feature was incorporated into wage and price setting, and into a medium scale DSGE model that featured other frictions by Christiano et al (2004) and Smets and Wouters (2003).

However, a line of thought questioning the validity of indexation-induced instrinsic inflation persistence has developed. This line of thought begins by noting that inflation persistence differs across countries (Levin and Piger (2003)) and across monetary regimes (Benati, 2008). It further notes that indexation is incompatible with the micro-evidence on price setting. A crude consensus from that literature is that prices at the firm level do not change continuously, as predicted by the indexation model, but in discreet jumps, in between which they remain unchanged. (See, for example, Alvarez *et al* (2006)). Finally, Dittmar *et al* (2005) noted that reduced form inflation persistence can result even in a flexible price model if monetary policy is chosen appropriately, reinforcing the idea that this econometric evidence in and of itself was not enough to verify the sticky price indexation model.

The argument that inflation persistence is a function of the monetary regime - an argument that explains why persistence varies across countries and monetary regimes - begs the question why regimes might choose inflation to be persistent if there was no need for it to be. Our paper offers a tentative answer. Inflation is persistent because at some times and in some places policymakers have believed it to be so.

2 The Model

We sketch the models we use only briefly, and confine the bulk of our attention to what is different about this set up from those that preceed it. In essence, we take the familiar model of indexation, and imagine that the central bank is deluded into thinking it holds; we model a central bank such as this in 3 different, familiar price setting realities: Calvo (1983); an approximation to flexible prices which is Calvo (1983) but with the probability of resetting prices each period set to be very close to 1; and Juillard *et al* (2007).

2.1 Households

Our household set up is common to all three of the 'realities' that our deluded central bank operates in, and indeed the housholds in reality behave just as the central bank assumes them to.

There is a continuum of identical infinitely lived households with access to complete financial markets whose objective function is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Households have rational expectations. In particular, they know that the central bank is operating under the wrong model of the economy.

The period budget constraint faced by the representative household in t=0,1,2,... is

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t + J_t$$

and to ensure solvency we have

$$\lim_{T\to\infty} E_t \left[Q_{t,t+T} B_{t+T} \right] \geq 0$$

Since optimality requires maximizing the consumption index for any given expenditure level, we obtain

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

The period utility function used below is

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Thus the optimality conditions for the household are

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right]$$

and in their loglinearized form are

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t [c_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}]\}) \\ i_t &\equiv -\log Q_t + \log \beta \end{aligned}$$

Taking the Euler equation and imposing the equilibrium condition $y_t = c_t$ we obtain the dynamic IS curve, which can be expressed in terms of the "welfare-relevant" output gap, which is defined as follows. Letting y_t be output, define y_t^n to be the output level that would attain in the absence of nominal frictions and y_t^e to be the output level that would attain in the absence of nominal frictions and all other distortions. We define the output gap to be $\tilde{y}_t \equiv y_t - y_t^n$ and the welfare-relevant output gap to be $x_t \equiv y_t - y_t^e$. Clearly $\tilde{y}_t \equiv x_t + (y_t^e - y_t^n)$[1]. Thus, assuming constant technology ($\Delta y_t^e \equiv 0$) we obtain

$$x_t = E_t [x_{t+1}] - \frac{1}{\sigma} (i_t - E_t [\pi_{t+1}])$$

2.2 Firms

We examine equilibrium behaviour under three specifications of the "true" underlying model which differ only in terms of the firms' pricing protocols (the IS curve and policy rule are unchanged across specifications). We begin by describing the common core of our three specifications, before outlining the differing pricing assumptions. The descriptions will be brief and we refer the reader to the appendix for more thorough derivations.

There is a continuum of firms $i \in [0, 1]$, with each firm producing a differentiated good with an identical technology, assumed to take the form:

$$Y_t(i) = A_t f(N_t(i)) = A_t N_t(i)^{1-\alpha}$$

The variable cost of production is given by the wage bill

$$VC(i) \equiv W_t f^{-1} \left(\frac{Y_t(i)}{A_t} \right)$$

[1] In what follows I loosely interpret the gap between y_t^e and y_t^n to be due to time varying markups (see Gali () page ?) and assume that there exists a tax that ensures that there are no competitive distortions in the non-stochastic steady state. Note that this does not preclude competitive distortions but simply implies that they are zero in steady state.

Consequently, real marginal cost is given by

$$s_t(i) \equiv \frac{W_t}{P_t}\frac{1}{A_t}\Psi\left(\frac{Y_t(i)}{A_t}\right)$$

$$\Psi(y) \equiv \frac{1}{f'(f^{-1}(y))}$$

The period profit earned by firm i charging price $P_t(i)$ is, using the household labour supply FOC,

$$\Pi_t(i) = P_t(i)Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} - \frac{U_{n,t}}{U_{c,t}}P_t f^{-1}\left(\frac{Y_t(i)}{A_t}\right)$$

and, subject to pricing constraints to be discussed below, the firm attempts to maximize a suitably expected future stream of these period profits. Firms have rational expectations and forecast future inflation under full knowledge of the central bank's wrong model.

Now, following Calvo (1983) we assume that in any period each firm faces a certain probability, θ, that it will not be able to reoptimize its pricing structures. Given that a firm was last able to reoptimize in period t the price that prevails in future periods prior to the next reoptimization is to some extent beyond the control of the firm. The exact protocol for how a firm's price evolves between optimizations is the dimension along which our specifications of the true underlying economy differ.

2.2.1 Three Different Models of Price Setting

Our policy-maker wrongly believes that the true model of the economy is a Calvo pricing model with backward-looking price indexation. We consider three alternative realities for price setting - simple Calvo pricing without indexation, flexible prices and the model of Juillard et al (2007) where firms choose not just a new price when they re-optimise but also a price path slope, which determines how their price evolves when they are unable to optimise in future. Below we briefly outline each of the three models in turn.

Specification 1: Calvo Pricing The pricing protocol in our first specification that of the familiar basic New Keynesian framework with monopolistically competitive firms who, in any given period, have a certain probablity θ of being unable to reoptimize their price. This setup gives rise to the following aggregate supply relation, known as the New Keynesian Phillips Curve (NKPC), again expressed in terms of the welfare relevant output gap.

$$\pi = \kappa x_t + \beta E_t\left[\pi_{t+1}\right] + \varepsilon_t$$

Specification 2: Flexible Prices We choose to model flexible prices as a special case of Specification 1 with the probability of a firm being able to reset its price being set to a level just below 1. Numerically, this model behaves just as the flex price model, and approximating it this way and avoids having to build separate code to simulate this model under central bank learning.

Specification 3: Juillard et al (2007) While the previous pricing models are familiar within the literature, Juillard et al (2007) is less well known. Hence, we provide a brief description of the pricing structure as well as detailing the structural equations to which it gives rise.

Firms maximize the expectation of a suitably discounted stream of profits in the contingencies in which they are unable to reoptimize (determined by a Calvo mechanism). On the occasions when they are able to reoptimize, firms (indexed by i) not only choose a new price, $P_t(i)$, for the current period but also a "slope", $v_t(i)$, of the path taken by the firm's price until the next opportunity to reoptimize. Thus, if firm i last set its price and slope in time t then its price profile in future periods before the firm is next able to reoptimize is given by

$$\begin{aligned} P_{t,t+1}(i) &= P_t(i)\, v_t(i) \\ P_{t,t+k}(i) &= P_t(i)\, v_t^k(i) \end{aligned}$$

The above protocol leads to three (log-linearized) structural equations involving inflation, π_t, a slope variable v_t, and an inertial variable ψ_t that captures the effects of slopes chosen in previous periods by firms who have not yet been allowed to reoptimize[2]

$$\begin{aligned} E_t v_{t+1} &= \theta_{v1}\pi_t + \theta_{v2}v_t + \theta_{v3}x_t + \theta_{v4}\psi_t + \theta_{v5}\varepsilon_t \\ E_t \pi_{t+1} &= \theta_{\pi1}\pi_t + \theta_{\pi2}v_t + \theta_{\pi3}x_t + \theta_{\pi4}\psi_t + \theta_{\pi5}\varepsilon_t \\ \psi_t &= \theta_{\psi1}\psi_{t-1} + \theta_{\psi2}v_{t-1} \end{aligned}$$

We simulate using this pricing protocol because it has the nice feature that indexation is a function of monetary policy. It therefore seems ripe for generating self-confirming equilbria: if monetary policy starts out generating inflation persistence, then a model with indexation endogenous to monetary policy has the potential to confirm this. We use the other pricing protocols because it serves to illustrate that such equilibria can emerge without any endogeneity between nominal rigidity in price setting and monetary policy.

[2] WE slightly abuse the notation here with v_t now representing the log linearized slope

2.3 The Central Bank

2.3.1 The Policy Problem

The central bank is unaware that in reality prices are set according to the models set out above. It sets the interest rate, i_t, subject to his current beliefs and misperceived model of the economy:[3]

$$
\begin{aligned}
W_t &= \min_{\{i_{t+k}\}_{k=0:\infty}} E_t \sum_{k=0}^{\infty} \beta^k \left[(\pi_{t+k} - \hat{\gamma}_t \pi_{t+k-1})^2 + \lambda x_{t+k}^2 \right] \qquad (1) \\
& s.t. \\
\pi_t - \hat{\gamma}_t \pi_{t-1} &= \kappa x_t + \beta E_t \left[\pi_{t+1} - \hat{\gamma}_t \pi_t \right] + \varepsilon_t \\
x_t &= E_t \left[x_{t+1} \right] - \frac{1}{\sigma} \left(i_t - E_t \left[\pi_{t+1} \right] \right) \\
\varepsilon_t &= \rho \varepsilon_{t-1} + \eta_t \\
\eta_t &\backsim N\left(0, \sigma_\eta^2\right)
\end{aligned}
$$

Note that the minimization is subject to a correctly perceived dynamic IS curve, a correctly perceived stochastic process for distortions and an incorrectly perceived New Keynesian Phillips Curve amended for the effects of indexation. In addition, note that the quasi-differencing parameter used by the central bank in its loss function is its current indexation parameter belief, $\hat{\gamma}_t$. Although none of the model specifications we consider feature indexation, the central bank behaves as if it is operating within a NKI model with a fixed γ equal to its current belief, $\hat{\gamma}_t$. Note that we adopt the assumption of "anticipated utility", which implies that the central bank behaves as if its current belief, $\hat{\gamma}_t$, will be unchanged in the future despite the fact that, as discussed in the next section, this belief will evolve according to data realizations; and note too that the central bank treats its current estimate of γ as if it were instead known with certainty.

As shown in the appendix, one can derive the rule that the central bank follows when setting rates in terms of the states it observes at the beginning of the period $(\pi_{t-1}, \varepsilon_t)$. This representation takes the form

$$
i_t = \theta_1 (\hat{\gamma}_t) \pi_{t-1} + \theta_2 (\hat{\gamma}_t) \varepsilon_t \qquad (2)
$$

where θ_1 and θ_2 are functions of the parameters of the central bank's problem above and, thus, depend on γ.

Finally, note that in the equilibrium of the NKI model with indexation parameter equal to $\hat{\gamma}_t$ and with policy set according to (2), inflation evolves with the following structure

[3] We assume that policy is being undertaken under discretion.

$$
\begin{aligned}
\pi_t &= \hat{\gamma}_t \pi_{t-1} + \lambda q \varepsilon_t \quad (3) \\
q &\equiv \frac{1}{\lambda (1 - \beta\rho) + \kappa^2}
\end{aligned}
$$

2.3.2 Central Bank Learning

Following Gaspar *et al* (2006) we recognize that, as indicated by (3), inflation should evolve as an AR(1) process under optimal discretionary policy in the NKI model. Consequently, in Step 5 above, we posit that the central bank updates its γ belief using a recursive algorithm based on this form. Specifically, we assume the central bank uses constant gain, stochastic gradient learning (Evans and Honkapohja (2001)). This implies that

$$
\begin{aligned}
\hat{\gamma}_{t+1} &= \hat{\gamma}_t + \phi \pi_{t-1} (\tilde{\pi}_t - \hat{\gamma}_t \pi_{t-1}) \quad (4) \\
\tilde{\pi}_t &\equiv \pi_t - \lambda q \varepsilon_t \quad (5)
\end{aligned}
$$

where ϕ represents the constant gain.

This model is equivalent to a least squares learning model where the precision matrix R is set equal to the identity every period. That model is popular because under decreasing gain it typically converges to rational expectations. Here, since we are modelling a central bank misperception, convergence to RE is not relevant. We opted for stochastic gradient learning also for practical reasons. It is often found in learning applications that simulation of the recursion for R_t runs into numerical problems - R_t can become singular easily - and we found the same problem. As a final remark on this feature of the model, note that the R matrix serves the same purpose as $(X'X)^{-1}$ in a least squares regression. So omiiting it is like ommitting the correction for precision.

Note that we assume that the central bank sees the shock to the desired mark-up ε_t and therefore it makes forecast errors not because of this but because it persists in believing in the indexation model even though this model is incorrect.

2.4 Timing Protocol

We assume that the timing protocol within a given period is

- Step 1:

  Period t begins. At this point the previous period's inflation rate, π_{t-1}, is known to all agents and the central bank has a given belief, $\hat{\gamma}_t$.

- Step 2:

  A shock, ε_t, to the underlying distortions in the economy occurs and is known to all agents.

- Step 3:

  Aware of the economy's state $s_t = (\pi_{t-1}, \varepsilon_t)$ the policymaker sets the interest rate according to his misspecified model and belief, $\hat{\gamma}_t$.

- Step 4:

  The private sector makes its consumption and pricing decisions, resulting in realized aggregate inflation, π_t, and output, y_t.

- Step 5:

  Observing the data realizations, the central bank updates its belief to $\hat{\gamma}_{t+1}$.

- Step 6:

  Period t ends.

2.5 Model Solution Method

We assume a fully rational private sector. They take rational expectations over the central bank's misperceptions and its perpetual learning. The resultant model is nonlinear and this prevents us using standard linear rational expectations solvers to characterize the equilibrium. We solve the model using the parameterized expectations algorithm (PEA) developed by, among others, den Haan and Marcet (1990). In this approach the conditional expectation functions are approximated by parametric functions with the values of the parameters being determined by the requirement that they are consistent with the data they induce. We use a variant of the PEA advocated by Maliar and Maliar (2003). Their method incorporates moving bounds for the values of endogenous variables in the iterations of the PEA. This is a way to enhance the convergence properties of the model without relying on a good initial guess for the parameters of the expectations functions. The moving bounds involve imposing initially tight bounds for permissable realizations of endogenous variables and gradually widening them if the bounds are hit. The bounds prevent explosive paths that result from the badly misspecified initial belief paramterizations from hijacking the search for the correct equilibrium[4].

[4] Note that the algorithm below defines a naive application of the PEA in that it is advisable to repeat Steps 2 and 3 to obtain convergence under many different sets of shocks. The rationale for this is that we want to ensure

Algorithm 1 *PEA with Bounds*

Step1 (Functional Form)

1a Choose parametric functional forms to approximate the expectations functions: $\zeta(\cdot;\omega) : R^{nstate} \rightarrow R^{ne}$

Step2 (Initialization)

2a Initialize the parameters, of the approximate expectations function, ω^0

2b Initialize the central bank's initial beliefs, $\hat{\gamma}^0$

2c Choose initial bounds for endogenous variables, $(z^0_{\min}, z^0_{\max}) \in R^{njump+nend} \times R^{njump+nend}$

Step3 (PEA Simulation Loop)

3a Set $i = 1$

3b Draw $nsim$ structural shocks from the appropriate distributions

3c Simulate the economy for $nsim$ periods by combining the structural equations, the shock sequence, the bounds $\left(z^{i-1}_{\min}, z^{i-1}_{\max}\right)$ and the approximate expectations functions under parameterization ω^{i-1}.

3d Using this data, estimate the parameters of the approximate expectations functions, call this estimate $\hat{\omega}$.

3e Update the parameterization using a convex combination of the new estimate and the previous parameter vector: $\omega^i = \mu\hat{\omega} + (1-\mu)\,\omega^{i-1}$.

3f Note which bounds in $\left(z^{i-1}_{\min}, z^{i-1}_{\max}\right)$ were struck during the simulation and widen them accordingly to obtain $\left(z^{i}_{\min}, z^{i}_{\max}\right)$[5].

3g If $\rho\left(\omega^i, \omega^{i-1}\right) < tol$ for some metric ρ, then stop and record ω^i. Otherwise continue.

3h If i exceeds a pre-specified iterations limit then abandon this attempt at PEA convergence and record the failure to converge. Otherwise, set $i = i + 1$ and continue iterating through the loop.

---

that the various PEA loops with different random draws lead to approximately the same parameterization of the expectations functions and moments of endogenous varaibles. Intuitively, this is because we want to be confident that we have found an equilibrium, since, provided that the equilibrium is unique, all of the PEA algorithms should converge to approximately the the same point.

[5] If, say, x_t struck $x^{i-1}_{\max}$ during the simulations, flag this occurrence and set $x^i_{\max} = x^{i-1}_{\max} + \varphi\left\|x^{i-1}_{\max}\right\|$ for $\varphi > 1$

2.6

We calibrate our model and set parameters to the following values:

Parameter	**Interpretation**	**Value**
α	Exponent on labour in production function	0.66
β	Discount factor	0.99
ζ	Price elasticity of substitution	10
θ	Probability of not being able to reoptimize	0.7
σ	Coefficient of Relative Risk Aversion	2
ξ	Inverse of Frisch Elasticity of Labour Supply	1
λ	Weight on output gap in policymaker's loss function	0.5
σ_{ε}	S.D. of shock to underlying distortions	0.01
$\hat{\gamma}_0$	Initial central bank belief	?
$\hat{\chi}_0$	Initial central bank intercept coefficient belief	0
ϕ	Gain	0.02

In all cases we are guided by previous authors who have used models of this form. [Citations to follow in later draft]. We found it necessary to choose a lowish value for the gain in order to ensure reliable convergence of our PEA. But this value is not far off the 0.03 value used by, for example, Orphanides and Williams, in the context of private sector learning. The initial central bank belief is left blank in the table: in our simulations below we experiment with a variety of initial beliefs. In our baseline results reported here we choose to impose a non-microfounded value for the weight on the output gap in the policymaker's loss function. But we found that adopting a much lower microfounded weight leaves the qualitative results we report below unchanged, and has little quantitative effect either.

3 Results

With the learning process and optimal policy as specified above, we found that (i) $\hat{\gamma}_t$ was quickly learned to its boundary value of 1 in all models of price setting; (ii) it was difficult to get the PEA algorithm to converge; (iii) the equilibrium in all models of price setting generated positive steady state inflation. This itself is indicative of the capacity for self-confirming equilibria to emerge in a model of central bank learning, where the central bank is deluded in the way we constrain them to be. However, we diagnosed these outcomes as being the result of our central bank misattributing inflation persistence for the positive steady state inflation that emerges in equilibrium. That the central bank would persist in being so deluded seemed unappealling and tantamount to rigging the model in favour of getting self-confirming inflation persistence. For this reason, we implemented the following model for central bank learning and optimal policy:

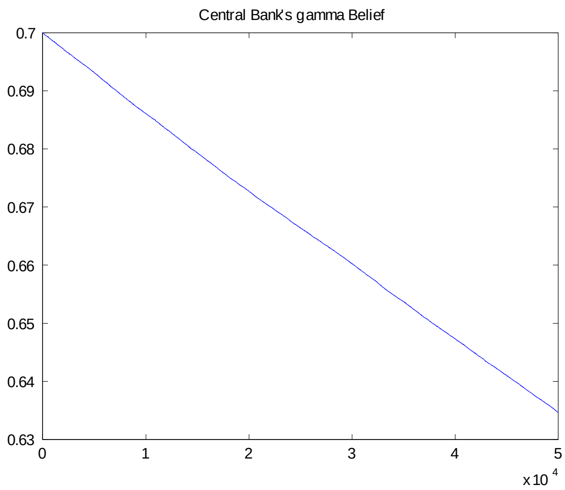
$$i_t = \frac{\widehat{\chi}_t}{1-\widehat{\gamma}_t} + \theta_1\left(\widehat{\gamma}_t\right)\pi_{t-1} + \theta_2\left(\widehat{\gamma}_t\right)\varepsilon_t \tag{6}$$

$$\begin{bmatrix} \widehat{\chi}_{t+1} \\ \hat{\gamma}_{t+1} \end{bmatrix} = \begin{bmatrix} \widehat{\chi}_t \\ \hat{\gamma}_t \end{bmatrix} + \phi \begin{bmatrix} 1 \\ \pi_{t-1} \end{bmatrix} \left(\tilde{\pi}_t - \widehat{\chi}_t - \hat{\gamma}_t \pi_{t-1}\right) \tag{7}$$

These equations include an intercept in the central bank's inflation equation, and a corresponding intercept in the interest rate rule. In all the equilibria that we discuss below, these intercepts are learned to zero. So the *equilibrium* policy rules and perceived laws of motion for inflation are as stated earlier.

3.1 Calvo

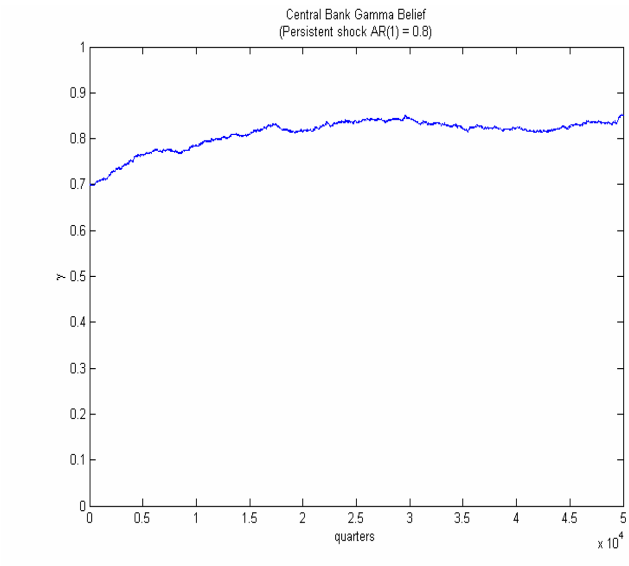
Our first set of results is where firms set prices according to the Calvo (1983) model. The chart below initialises central bank beliefs about the (non existent) indexation parameter at $\hat{\gamma}_0 = 0.7$. We see that there is a very slow but steady learning of $\hat{\gamma}_t$ down: the central bank's estimate falls by a little under 0.07 in 50k periods.



$\hat{\gamma}_0 = 0.7$, Calvo pricing

The same profile is observed for other initial central bank beliefs.

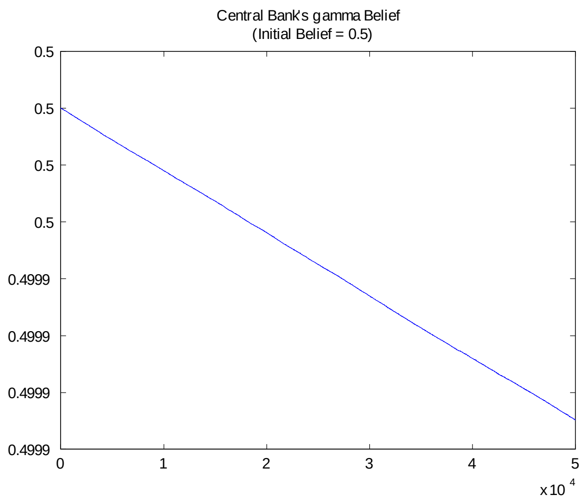
Next, we inject persistence into the mark-up shock, still with Calvo price-setting. Note that the policymaker knows the persistence of the mark-up process, (set to 0.8). The chart below reports what we find, starting from the same initial belief as before at $\hat{\gamma}_0 = 0.7$

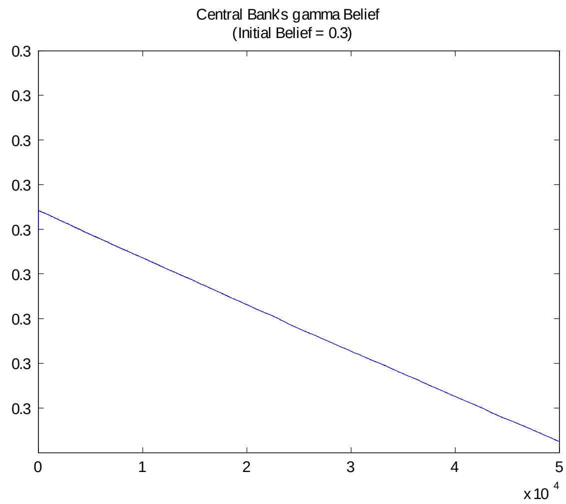


Once again we get self-confirming persistence of sorts. This time there is no slow drifting down of $\hat{\gamma}$. However, a caveat: relative to the other settings, we found it was particularly hard to get convergence of the PEA in this setting. For many draws of shocks, we don't get convergence.

3.2 Flexible Prices

We turn next to the results with the central bank learning about indexation in an approximation to the flexible price model: the Calvo model with the probability of prices being reset equal almost to 1. Here we also see a steady learning down of the value of central bank's indexation estimate $\hat{\gamma}$. This time the learning proceeds far more slowly than before, however. After 50k periods, the learned value of $\hat{\gamma}$ has fallen by less than 0.0001. This holds for regardless of initial beliefs ($\hat{\gamma}_0 = 0.5, 0.3$ reported below).

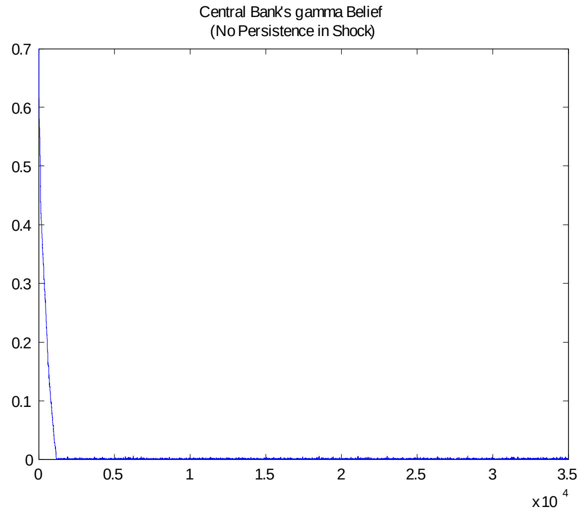




These simulations make contact with the work of Dittmar *et al* (2005) cited in the introduction. They show that it is possible for a monetary policymaker to generate reduced form inflation persistence if monetary policy is set appropriately. That work prompts two questions: is it possible to generate evidence of structural inflation persistence by appropriate chocie of monetary policy? And even if so, why would a monetary policymaker follow a monetary policy that was so designed? Our simulations provide an answer: if the central bank is trapped in the belief that the indexation model prevails, and sets monetary policy accordingly, small sample estimates of the data will confirm this wrong belief even under flexible prices.

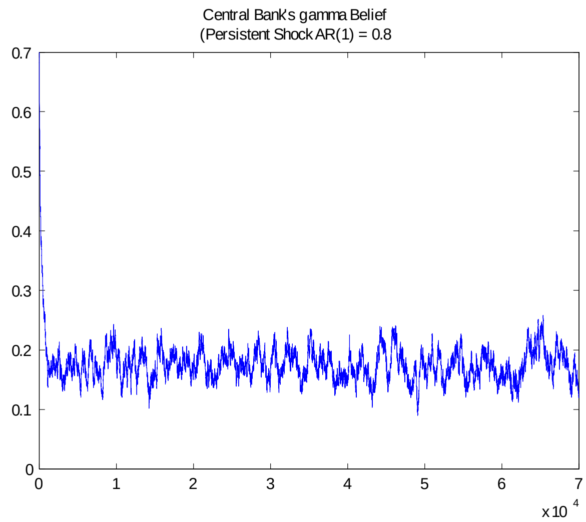
3.3 Juillard et al (2007)

Finally we present our results where the central bank operates in a world where prices are set according to Juillard *et al* (2007). Recall this is the model where each period that firms get to reset prices, they also get to reset the linear function they use to index prices for future periods when they are unable to re-optimise. We present two variants of this environment. In the first variant, just as with our simulations under Calvo and flexible pricing, there is no persistence in the mark-up shock.



In this simulation, we observe that the central bank's beliefs about indexation $\hat{\gamma}$ fall to 0 much faster than in the other pricing models. $\hat{\gamma}$ falls from an initial value of 0.7 to 0 within 2k periods. 2k periods is still a long time, however, if we take the quarterly frequency of the model calibration literally, so although inflation persistence is not strictly self confirming here either, it is taking a long time (500 years!) to eradicate beliefs in intrinsic inflation persistence.

In the second experiment using Juillard *et al* pricing, we make the mark-up shock follow and AR(1) with autoregressive coefficient of 0.8. We assume that policymakers know the process governing the shock precisely.



Here the equilibrium is quite different. From an initial value of 0.7, $\hat{\gamma}$ falls to and then fluctuates around a value around 0.18. In this model, inflation persistence is self-confirming and evolving around a low value. To repeat, the policymaker knows the persistence of the shock process here, so this self-confirmation is not coming about because we are introducing persistence into inflation that it cannot see.

4 Conclusions

Some controversy has arisen about whether there is intrinsic persistence in inflation due to the fact that firms may index prices. Whether there is or not has implications for how monetary policy responds to shocks. Indexation implies that monetary policy should be more accommodative. It has been observed that estimates of inflation persistence derived either from reduced-form or structural econometrics are somewhat regime-dependent, strong circumstancial evidence that such persistence is a product of the regime, and not intrinsic. That evidence begs the question why policy might at some time and in some countries choose higher inflation persistence and at some time choose low inflation persistence. In this paper we have shown that if a central bank starts out with the misconception that there is indexation, and engages in perpetual learning about this coefficient, then it can find its false initial beliefs effectively self-confirming. We have shown that this statement holds in a variety of settings: Calvo, flexible prices, and Juillard et al (2007). We qualify with 'effectively' because we find that in many settings, beliefs that inflation is intrinsically persistent are eventually learned away, but only very slowly. Under Calvo it takes about 50k periods to reduce an initial estimate of inflation persistence by 0.07; under flexible prices such estimates fall by less than 0.0001 over the same period. When indexation is a function of monetary policy, as in Juillard et al (2007), if shocks are not persistent, central bank beliefs converge on zero inflation persistence within 2k periods. If shocks are persistent, beliefs settle and fluctuate around a value of 0.2.

Two questions we have not so far addressed are these. We have shown that given an initial belief that inflation is persistent, inflation may be believed to be so for some time after that. But this begs the question how the belief that inflation is persistent arises in the first place. Second, we have so far not said anything about the welfare consequences of the self-confirmation in false beliefs about inflation persistence. We confine ourselves here to remarking that the welfare implications will of course depend on what model explains price setting in reality. If prices are sticky in the fashion posited by Calvo (1983), the welfare consequences of setting policy conditional on a belief that there is indexation may be quite severe. The overly accommodative response to shocks will generate price dispersion that is costly. If, however, prices are flexible, the process for inflation chosen will matter less for welfare. In the cashless limits studied in this paper, the costs of choosing the wrong inflation process under flexible prices will be infinitessamlly small. In more realistic models that articulate money demand, there will be welfare consequences. A volatile inflation rate will induce lower welfare.

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Part I

Appendix

In this appendix, we recap on the derivation of the Philips Curves under our three different models of price-setting, and in the model of indexation that characterises the central bank's delusion about our model worlds. We also recap on the derivation of optimal monetary policy under indexation.

6 A: The New Keynesian Indexation Model

6.1 Households

There is a continuum of identical infinitely lived households whose objective function is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The period budget constraint faced by the representative household in t=0,1,2,... is

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + J_t$$

and to ensure solvency we have

$$\lim_{T \to \infty} E_t \left\{ Q_{t,t+T} B_{t+T} \right\} \geq 0$$

Since optimality requires maximizing the consumption index for any given expenditure level, we obtain

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

Consequently, one can reexpress the period budget constraint as

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + J_t$$

The period utility function used below is

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Thus the optimality conditions for the household are

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

and in their loglinearized form are

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} \right) \end{aligned}$$

$$i_t \equiv -\log Q_t + \log \beta$$

6.2 Firms

There is a continuum of firms $i \in [0,1]$, with each firm producing a differentiated good with an identical technology, assumed to take the form:

$$Y_t(i) = A_t f(N_t(i)) = A_t N_t(i)^{1-\alpha}$$

The variable cost of production is given by the wage bill

$$VC(i) \equiv W_t f^{-1}\left(\frac{Y_t(i)}{A_t}\right)$$

Consequently, real marginal cost is given by

$$s_t(i) \equiv \frac{W_t}{P_t}\frac{1}{A_t}\Psi\left(\frac{Y_t(i)}{A_t}\right)$$

$$\Psi(y) \equiv \frac{1}{f'(f^{-1}(y))}$$

6.2.1 Flexible Price Case

It is instructive to consider the case of flexible prices due to its relevance for the steady state of the indexation model. The markup, μ, instituted by a monopolistically competitive firm facing an isoelastic demand schedule with elasticity ε is shown below.

$$\frac{P_t(i)}{P_t} = \mu.s_t(i)$$

$$\mu \equiv \frac{\varepsilon}{\varepsilon - 1}$$

Combining this with the demand constraints faced by the firms, we obtain

$$\left(\frac{Y_t(i)}{Y_t}\right)^{-\frac{1}{\varepsilon}} = \mu.s_t(i)$$

Since, in equilibrium, we have all firms producing the same quantity, equal to aggregate production, we observe that marginal cost is then the inverse of the markup ($\bar{s} = \mu^{-1}$). In the indexation model, this will be the case in steady state.

6.2.2 Optimal Price Setting

The period profit earned by firm i charging price $P_t(i)$ is, using the household labour supply FOC,

$$\Pi_t(i) = P_t(i) Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} - \frac{U_{n,t}}{U_{c,t}} P_t f^{-1}\left(\frac{Y_t(i)}{A_t}\right)$$

For a firm unable to reoptimze its price in period t+j, the price of its product will be that in period t+j-1 multiplied by (gross) inflation in period t+j-1 raised to the power γ. Consequently, in the case of a firm that last reoptimized in period t, its price in period t+j is given by

$$P_{t+k}(i) = P_t(i)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma}$$

The stochastic discount factor for evaluating nominal payoffs is given by

$$Q_{t,t+k} \equiv \beta^k \frac{U_{c,t+k}}{U_{c,t}} . \frac{P_t}{P_{t+k}}$$

Thus the firm's objective function is

$$\max_{P_t(i)} E_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \Pi_{t+k}\right]$$

The FOC for this problem can be expressed as follows (compare to Gali (9) Ch.3):

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t}\left[P_t^*\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma} - \mu P_{t+k} s_{t+k|t}\right]$$

Dividing through by P_{t-1} and log-linearizing around a zero inflation steady state we obtain (lower case indicates log prices)

$$\sum_{k=0}^{\infty} (\theta\beta)^k \left(p_t^* - p_{t-1}\right) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t\left(\hat{s}_{t+k|t} + (p_{t+k} - p_{t-1}) - \gamma\left(p_{t+k-1} - p_{t-1}\right)\right)$$

Re-expressed (do lots of adding and subtracting of terms here) we have:

$$p_t^* - p_{t-1} = (1-\theta\beta)\sum_{k=0}^{\infty} (\theta\beta)^k E_t\left(\hat{s}_{t+k|t} + \sum_{s=1}^{k}\left(\pi_{t+s} - \gamma\pi_{t+s-1}\right) + \pi_t\right)$$

or

$$p_t^* - p_t = (1-\theta\beta)\sum_{k=0}^{\infty} (\theta\beta)^k E_t\left(\hat{s}_{t+k|t} + \sum_{s=1}^{k}\left(\pi_{t+s} - \gamma\pi_{t+s-1}\right)\right)$$

Now, (see Gali Ch.3 P.7) we can express the log real marginal cost of a firm in $t+k$, which last reoptimized in t, in terms of the log average real marginal cost of the economy and a term related to the deviation of the firm's price from the average price level:

$$\begin{aligned}\log s_{t+k|t} &= \log s_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* + \gamma\left(p_{t+k-1} - p_{t-1}\right) - p_{t+k}\right) \\ &= \log s_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_t - \sum_{s=1}^{k}\left(\pi_{t+s} - \gamma\pi_{t+s-1}\right)\right)\end{aligned}$$

Thus we have

$$p_t^* - p_t = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\hat{s}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left((p_t^* - p_t) - \sum_{s=1}^{k}\left(\pi_{t+s} - \gamma\pi_{t+s-1}\right)\right) + \sum_{s=1}^{k}\left(\pi_{t+s} - \gamma\pi_{t+s-1}\right)\right)$$

which implies

$$\begin{aligned}p_t^* - p_t &= (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\Theta\hat{s}_{t+k} + \sum_{s=1}^{k}\pi_{t+s}^{QD}\right) \\ \Theta &\equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \\ \pi_{t+s}^{QD} &\equiv \pi_{t+s} - \gamma\pi_{t+s-1}\end{aligned}$$

More usefully, the above can be re-expressed as

$$p_t^* - p_{t-1} - \gamma\pi_{t-1} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left(\Theta\hat{s}_{t+k} + \sum_{s=0}^{k}\pi_{t+s}^{QD}\right)$$

6.2.3 Combining with Price Evolution

Under the pricing structure described above, the aggregate price level evolves according to

$$P_t = \left[(1-\theta)(P_t^*)^{1-\varepsilon} + \theta\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

In the second term on the RHS above, the lagged aggregate price index P_{t-1} appears before the scaled gross inflation rate by a Law of Large Numbers argument. Given this expression for the evolution of the price index, one can log-linearize around the zero inflation steady state to obtain

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) + \theta\gamma\pi_{t-1}$$

Thus

$$p_t^* - p_{t-1} - \gamma \pi_{t-1} = \frac{\pi_t^{QD}}{1-\theta}$$

Therefore we have

$$\begin{aligned} \frac{\pi_t^{QD}}{1-\theta} &= (1-\theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(\Theta \hat{s}_{t+k} + \sum_{s=1}^{k} \pi_{t+s}^{QD} \right) \\ &= (1-\theta\beta) \Theta \hat{s}_t + \pi_t^{QD} + (1-\theta\beta) \sum_{k=1}^{\infty} (\theta\beta)^k E_t \left(\Theta \hat{s}_{t+k} + \sum_{s=1}^{k} \pi_{t+s}^{QD} \right) \end{aligned}$$

Consequently

$$\pi_t^{QD} \frac{\theta}{1-\theta} = (1-\theta\beta) \Theta \hat{s}_t + \theta\beta E_t \frac{\pi_{t+1}^{QD}}{1-\theta}$$

So

$$\pi_t^{QD} = \beta E_t \pi_{t+1}^{QD} + \frac{(1-\theta)(1-\theta\beta)\Theta}{\theta} \hat{s}_t$$

6.2.4 Marginal Cost vs Output Deviations

Now, in order to express the economy's average real marginal cost deviation in terms of output deviations:

$$\begin{aligned} \log(s_t) &= w_t - p_t - mpn_t \\ &= (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \\ \log(\bar{s}) &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \end{aligned}$$

In the last line above, y_t^n, represents the natural level of output that would attain under flexible prices. As aforementioned, real marginal cost under flexible prices is equal to the inverse of the desired mark-up, μ. Consequently we obtain

$$y_t^n = -\psi_{y,0}^n + \psi_{y,a}^n a_t$$

$$\hat{s}_t = \zeta(y_t - y_t^n)$$

$$\begin{aligned}
\psi^n_{y,0} &\equiv \frac{(1-\alpha)(\log\mu - \log(1-\alpha))}{\sigma+\varphi+\alpha(1-\sigma)} \\
\psi^n_{y,1} &\equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} \\
\zeta &\equiv \sigma + \frac{\varphi+\alpha}{1-\alpha}
\end{aligned}$$

Thus we can express the log deviation of real marginal cost from steady state in terms of the deviation in output from its natural level.

6.3 NKPC Under Indexation

Consequently we have the NKPC under indexation

$$\pi_t^{QD} = \kappa\tilde{y}_t + \beta E_t \pi_{t+1}^{QD}$$

$$\begin{aligned}
\kappa &\equiv \frac{(1-\theta\beta)(1-\theta)\Theta}{\theta}\zeta \\
\tilde{y}_t &\equiv y_t - y_t^n
\end{aligned}$$

To obtain the welfare-relevant output gap representation we use the identities

$$x_t \equiv y_t - y_t^e$$

$$\tilde{y}_t \equiv x_t + (y_t^e - y_t^n)$$

where y_t^e is the efficient level of output (which would be attained under flexible prices in the absence of market power distortions).

Consequently one obtains

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \beta E_t\left[\pi_{t+1} - \gamma\pi_t\right] + \varepsilon_t^{PC}$$

$$\varepsilon_t^{PC} \equiv \kappa\left(y_t^e - y_t^n\right)$$

We assume that ε_t^{PC} follows an AR(1) process:

$$\varepsilon_t^{PC} = \rho^{PC}\varepsilon_{t-1}^{PC} + v_t^{PC}$$

Note, that the "normal" Calvo model can be obtained by setting $\gamma = 0$.

7 B: The Juillard *et al* Model

7.1 Households

The behavious of households is the same as in the NKI model above and thus the corresponding derivations are omitted here.

7.2 Firms

As above, firms maximize the expectation of a suitably discounted stream of profits in the contingencies in which they are unable to reoptimize fully. However, unlike in the indexation model above, firms in this model do not update their prices according to lagged aggregate inflation whenever they are unable to reoptimize fully. Instead, on the occasions when they do reoptimize, firms not only choose a new price, $P_t(i)$, but also an "individual (gross) inflation rate", $v_t(i)$, at which this price will be updated until the next opportunity to reoptimize. The period profit for firm i who last reoptimized in period t is thus

$$\Pi_{t+k}(i) = P_t(i)v_t^k Y_{t+k}\left(\frac{P_t(i)v_t^k}{P_{t+k}}\right)^{-\varepsilon} - \frac{U_{n,t+k}}{U_{c,t+k}}P_{t+k}f^{-1}\left(\frac{Y_{t+k}\left(\frac{P_t(i)v_t^k(i)}{P_{t+k}}\right)^{-\varepsilon}}{A_{t+k}}\right)$$

The FOCs for optimal choice of $P_t(i)$ and $v_t(i)$ are then

$$E_t\sum_{k=0}^{\infty}(\theta\beta)^k\frac{U_{c,t+k}}{U_{c,t}}\left(\frac{P_t(i)v_t^k(i)}{P_{t+k}}\right)^{-\varepsilon}Y_{t+k}\left[\frac{P_t(i)v_t^k(i)}{P_{t+k}} - \mu s_{t+k}\right] = 0$$

$$E_t\sum_{k=0}^{\infty}(\theta\beta)^k\frac{U_{c,t+k}}{U_{c,t}}k\left(\frac{P_t(i)v_t^k(i)}{P_{t+k}}\right)^{-\varepsilon}Y_{t+k}\left[\frac{P_t(i)v_t^k(i)}{P_{t+k}} - \mu s_{t+k}\right] = 0$$

Noting that all firms reoptimizing in t will choose the same control values one can rearrange the above FOCs to obtain

$$\begin{aligned} p_t &= \frac{E_t\left[\sum_{k=0}^{\infty}\omega\left(k,t\right)\mu s_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty}\frac{P_t}{P_{t+k}}v_t^k\omega(k,t)\right]} \\ p_t &= \frac{E_t\left[\sum_{k=0}^{\infty}\omega\left(k,t\right)\mu s_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty}k\frac{P_t}{P_{t+k}}v_t^k\omega(k,t)\right]} \end{aligned}$$

where (with a slight change in notation)

$$p_t \equiv \frac{P_t^*}{P_t}$$

$$\omega(k,t) \equiv (\theta\beta)^k \frac{u_{c,t+k}}{u_{c,t}} \left(\frac{P_t^* v_t^k}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$$

Reexpressing these FOCs in a more compact form:

$$\begin{aligned}
p_t &= \frac{E_t\left[\sum_{k=0}^{\infty}\omega\left(k,t\right)\mu s_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty}\frac{v_t^k}{\Pi_{t,k}}\omega(k,t)\right]} \\
p_t &= \frac{E_t\left[\sum_{k=0}^{\infty}\omega\left(k,t\right)\mu s_{t+k}\right]}{E_t\left[\sum_{k=0}^{\infty}k\frac{v_t^k}{\Pi_{t,k}}\omega(k,t)\right]} \\
\Pi_{t,k} &\equiv \prod_{s=0}^{k-1}\left(\frac{P_{t+s+1}}{P_{t+s}}\right)
\end{aligned}$$

Log-linearizing, one obtains

$$\begin{aligned}
\frac{\hat{p}_t}{1-\theta\beta} + \frac{\hat{v}_t\theta\beta}{(1-\theta\beta)^2} &= E_t\sum_{k=0}^{\infty}(\theta\beta)^k\left[\hat{s}_{t+k}+\hat{\Pi}_{t+k}\right] \\
\hat{p}_t\frac{\theta\beta}{(1-\theta\beta)^2} + \hat{v}_t\frac{\theta\beta\left(1+\theta\beta\right)}{(1-\theta\beta)^3} &= E_t\sum_{k=0}^{\infty}(\theta\beta)^k k\left[\hat{s}_{t+k}+\hat{\Pi}_{t+k}\right]
\end{aligned}$$

Concentrating first on the "price" FOC we multiply by $(1-\theta\beta)$ and then take expectations of the next period version, multiplied by $(1-\theta\beta)^2$.

$$\hat{p}_t + \hat{v}_t\frac{\theta\beta}{1-\theta\beta} = \left(1-\theta\beta\right)E_t\sum_{k=0}^{\infty}(\theta\beta)^k\left[\hat{s}_{t+k}+\hat{\Pi}_{t,k}\right] \quad (8)$$

$$\left(1-\theta\beta\right)E_t\hat{p}_{t+1} + \theta\beta E_t\hat{v}_{t+1} = \left(1-\theta\beta\right)^2 E_t\left[\sum_{k=0}^{\infty}(\theta\beta)^k\hat{s}_{t+1+k} + \sum_{k=1}^{\infty}(\theta\beta)^k\hat{\Pi}_{t+1,k}\right] \quad (9)$$

Note the summation from k=1 in the square brackets above is correct since

$$\begin{aligned}
\hat{\Pi}_{t+1,k} &= 0 && k=0 \\
&= \hat{\pi}_{t+2}^G && k=1 \\
&= \hat{\pi}_{t+2}^G + ... + \hat{\pi}_{t+k+1}^G && k=2,3,...
\end{aligned}$$

Also, note

$$\begin{aligned}
\hat{\pi}_t^G &\equiv \log\left(1+\pi_t\right) - \log 1 \\
&= \log\left(1+\pi_t\right) \\
&\approx \pi_t
\end{aligned}$$

By definition we therefore have

$$\hat{p}_t + \hat{v}_t \frac{\theta\beta}{1-\theta\beta} = (1-\theta\beta) E_t \left[\hat{s}_t + (\theta\beta)(\hat{s}_{t+1} + \pi_{t+1}) + (\theta\beta)^2 (\hat{s}_{t+2} + \pi_{t+1} + \pi_{t+2}) + ...\right] \quad (10)$$

$$(\theta\beta) E_t [\hat{p}_{t+1}] + \frac{(\theta\beta)}{1-\alpha\beta} E_t \hat{v}_{t+1} = (1-\theta\beta) E_t \left[(\theta\beta)\hat{s}_{t+1} + (\alpha\beta)^2 (\hat{s}_{t+2} + \pi_{t+2} + ...)\right] \quad (11)$$

Subtracting 11 from 10 yields

$$\hat{p}_t - E_t\hat{p}_{t+1} + (1-\theta\beta) E_t\hat{p}_{t+1} + \frac{\theta\beta}{1-\theta\beta} [\hat{v}_t - E_t\hat{v}_{t+1} + (1-\theta\beta) E_t\hat{v}_{t+1}] = (1-\theta\beta)\hat{s}_t + \theta\beta E_t\pi_{t+1}$$

Therefore

$$\theta\beta E_t\hat{p}_{t+1} - \hat{p}_t + \frac{\theta\beta}{1-\theta\beta} [E_t\hat{v}_{t+1} - \hat{v}_t] = -(1-\theta\beta)(\hat{s}_t + \theta\beta E_t\hat{v}_{t+1} - \theta\beta E_t\pi_{t+1}) \quad (12)$$

Rearranging 8 and then taking expectations of the next period version, multiplied by $(1-\theta\beta)^2$ we have

$$\hat{p}_t + \frac{1+\theta\beta}{1-\theta\beta}\hat{v}_t = \frac{(1-\theta\beta)^2}{\theta\beta} E_t \left[(\theta\beta)(\hat{s}_{t+1} + \pi_{t+1}) + 2(\theta\beta)^2 (\hat{s}_{t+2} + \pi_{t+1} + \pi_{t+2}) + ...\right] \quad (13)$$

$$\theta\beta E_t\hat{p}_{t+1} + \frac{\theta\beta(1+\theta\beta)}{1-\theta\beta} E_t\hat{v}_{t+1} = \frac{(1-\theta\beta)^2}{\theta\beta} E_t \left[(\theta\beta)^2 (\hat{s}_{t+2} + \pi_{t+2}) + 2(\theta\beta)^3 (\hat{s}_{t+3} + \pi_{t+2} + \pi_{t+3}) + ...\right] \quad (14)$$

Subtracting 14 from 13 and then re-arranging we obtain

$$\begin{aligned}
& \hat{p}_t - \theta\beta E_t\hat{p}_{t+1} + \frac{1+\theta\beta}{1-\theta\beta} [\hat{v}_t - \theta\beta E_t\hat{v}_{t+1}] \\
= \; & E_t\pi_{t+1} + (1-\theta\beta)^2 E_t \left[\hat{s}_{t+1} + (\theta\beta)(\hat{s}_{t+2} + \pi_{t+2}) + (\theta\beta)^2 (\hat{s}_{t+3} + \pi_{t+2} + \pi_{t+3}) + ...\right] \\
= \; & E_t\pi_{t+1} + (1-\theta\beta)^2 \left\{ E_t \sum_{k=0}^{\infty} (\theta\beta)^k \hat{s}_{t+1+k} + E_t \left[(\theta\beta)\pi_{t+2} + (\theta\beta)^2 (\pi_{t+2} + \pi_{t+3}) + ...\right] \right\} \\
= \; & E_t\pi_{t+1} + (1-\theta\beta)^2 E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^k \hat{s}_{t+1+k} + \sum_{k=1}^{\infty} (\theta\beta)^k \hat{\Pi}_{t+1,k} \right]
\end{aligned}$$

Recalling 9 we obtain

$$[\hat{p}_t - E_t\hat{p}_{t+1}] + \frac{1+\theta\beta}{1-\theta\beta} [\hat{v}_t - E_t\hat{v}_{t+1}] = E_t\pi_{t+1} - E_t\hat{v}_{t+1}$$

Alternatively expressed:

$$E_t\hat{p}_{t+1} = \hat{p}_t + E_t\hat{v}_{t+1} - E_t\pi_{t+1} - \frac{1+\theta\beta}{1-\theta\beta}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right] \tag{15}$$

$$E_t\hat{p}_{t+1} - \hat{p}_t = -\frac{2\theta\beta}{1-\theta\beta}E_t\hat{v}_{t+1} - E_t\pi_{t+1} + \frac{1+\theta\beta}{1-\theta\beta}\hat{v}_t \tag{16}$$

Combining 12 and 15 we obtain

$$\begin{aligned} & \theta\beta\left[\hat{p}_t + E_t\hat{v}_{t+1} - E_t\pi_{t+1} - \frac{1+\theta\beta}{1-\theta\beta}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right]\right] - \hat{p}_t + \frac{\theta\beta}{1-\theta\beta}\left[E_t\hat{v}_{t+1} - \hat{v}_t\right] \\ = & -(1-\theta\beta)\left(\hat{s}_t + \theta\beta E_t\hat{v}_{t+1} - \theta\beta E_t\pi_{t+1}\right) \end{aligned}$$

Rearrangement yields

$$E_t\hat{v}_{t+1} - \hat{v}_t = \left(\frac{1-\theta\beta}{\theta\beta}\right)^2 (\hat{s}_t - \hat{p}_t) \tag{17}$$

7.3 The Price Index

The evolution equation for the aggregate price index is

$$P_t = \left[(1-\theta)\sum_{s=0}^{\infty}\theta^s\left[P^*_{t-s}v^s_{t-s}\right]^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

This implies

$$\begin{aligned} \left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} = & (1-\theta)\left(\frac{P^*_t}{P_t}\right)^{1-\varepsilon}\left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} + (1-\theta)\,\theta\left(\frac{P^*_{t-1}}{P_{t-1}}\right)^{1-\varepsilon} v^{1-\varepsilon}_{t-1} \\ & + (1-\theta)\,\theta^2\left(\frac{P^*_{t-2}}{P_{t-2}}\right)^{1-\varepsilon}\left(\frac{P_{t-2}}{P_{t-1}}\right)^{1-\varepsilon}\left(v^2_{t-2}\right)^{1-\varepsilon} + ... \end{aligned}$$

or, alternatively expressed,

$$1 = (1-\theta)\,p^{1-\varepsilon}_t + (1-\theta)\,\theta p^{1-\varepsilon}_{t-1}\left(\frac{v_{t-1}}{\pi^G_t}\right)^{1-\varepsilon} + (1-\theta)\,\theta^2 p^{1-\varepsilon}_{t-2}\left(\frac{v^2_{t-2}}{\pi^G_t\pi^G_{t-1}}\right)^{1-\varepsilon} + ...$$

In log-linearized from we have

$$0 = \hat{p}_t + \theta\left(\hat{p}_{t-1} + \hat{v}_{t-1} - \pi_t\right) + \theta^2\left(\hat{p}_{t-2} + 2\hat{v}_{t-2} - \pi_{t-1} - \pi_t\right) + ...$$

Thus

$$\pi_t\left(\theta+\theta^2+\theta^3+...\right)=\hat{p}_t+\theta\left(\hat{p}_{t-1}+\hat{v}_{t-1}\right)+\theta^2\left(\hat{p}_{t-2}+2\hat{v}_{t-2}-\pi_{t-1}\right)+...$$

Equivalently we have

$$\begin{aligned}\pi_t &= \frac{1-\theta}{\theta}\hat{p}_t+(1-\theta)\left(\hat{p}_{t-1}+\hat{v}_{t-1}\right)+\theta(1-\theta)\left(\hat{p}_{t-2}+2\hat{v}_{t-2}-\pi_{t-1}\right) \\ &\quad +\theta^2(1-\theta)\left(\hat{p}_{t-3}+3\hat{v}_{t-2}-\pi_{t-2}-\pi_{t-1}\right)+...\end{aligned} \tag{18}$$

7.4 Quasi-Differencing

Quasi-differencing 18 yields

$$\begin{aligned}\pi_t-\theta\pi_{t-1} &= \frac{1-\theta}{\theta}\hat{p}_t+(1-\theta)\hat{v}_{t-1}+(1-\theta)\theta\left(\hat{v}_{t-2}-\pi_{t-1}\right)+(1-\theta)\theta^2\left(\hat{v}_{t-3}-\pi_{t-1}\right)+... \\ &= \frac{1-\theta}{\theta}\hat{p}_t-\theta\pi_{t-1}+(1-\theta)\hat{v}_{t-1}+(1-\theta)\theta\hat{v}_{t-2}+(1-\theta)\theta^2\hat{v}_{t-3}+...\end{aligned}$$

Consequently we have

$$\pi_t=\frac{1-\theta}{\theta}\hat{p}_t+\hat{\psi}_t \tag{19}$$

$$\hat{\psi}_t\equiv(1-\theta)\hat{v}_{t-1}+(1-\theta)\theta\hat{v}_{t-2}+(1-\theta)\theta^2\hat{v}_{t-3}+...$$

Clearly the "inertial" variable $\hat{\psi}_t$ evolves according to

$$\hat{\psi}_t=\theta\hat{\psi}_{t-1}+(1-\theta)\hat{v}_{t-1} \tag{20}$$

Note, for future reference,

$$E_t\psi_{t+1}-\psi_t=(1-\theta)\hat{v}_t-(1-\theta)\hat{\psi}_t \tag{21}$$

7.5 Final Inflation Dynamics

7.5.1 Slope Evolution Equation

Combining 17 and 19 we have

$$E_t\hat{v}_{t+1}=\hat{v}_t+\left(\frac{1-\theta\beta}{\theta\beta}\right)^2\hat{s}_t-\left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta}\pi_t+\left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta}\hat{\psi}_t \tag{22}$$

7.5.2 Inflation Equation ("Phillips Curve")

Using 19 we obtain

$$E_t\pi_{t+1} - \pi_t = \frac{1-\theta}{\theta}\left(E_t\hat{p}_{t+1} - \hat{p}_t\right) + E_t\hat{\psi}_{t+1} - \hat{\psi}_t$$

Combining this with 16, and 21 we have

$$E_t\pi_{t+1} - \pi_t = (1-\theta)\,\hat{v}_t - (1-\theta)\,\hat{\psi}_t + \frac{1-\theta}{\theta}\left[\frac{-2\theta\beta}{1-\theta\beta}E_t\hat{v}_{t|+1} - E_t\pi_{t+1} + \frac{1+\theta\beta}{1-\theta\beta}\hat{v}_t\right]$$

Using 22 to substitute for $E_t\hat{v}_{t+1}$ and rearranging we have

$$\begin{aligned}\frac{1}{\theta}E_t\pi_{t+1} &= \pi_t\left(1+\frac{2\,(1-\theta\beta)}{\theta\beta}\right) + \hat{v}_t\left(\frac{1-\theta}{\theta}\frac{1+\theta\beta}{1-\theta\beta} + 1 - \theta - \frac{1-\theta}{\theta}\frac{2\theta\beta}{1-\theta\beta}\right) \\ &\quad -\hat{\psi}_t\left(\frac{2\,(1-\theta\beta)}{\theta\beta} + 1 - \theta\right) - \hat{s}_t\left(\frac{1-\theta}{\theta}\frac{2\,(1-\theta\beta)}{\theta\beta}\right)\end{aligned}$$

Thus

$$\begin{aligned}E_t\pi_{t+1} &= \pi_t\left(\frac{2}{\beta}-\theta\right) + \hat{v}_t\left((1-\theta)\,(1+\theta)\right) + \hat{\psi}_t\left(\theta\,(1+\theta) - \frac{2}{\beta}\right) + \hat{s}_t\left(\frac{-2\,(1-\theta)\,(1-\theta\beta)}{\theta\beta}\right) \\ &= \pi_t\left(\frac{2}{\beta}-\theta\right) + \hat{v}_t\left((1-\theta)\,(1+\theta)\right) + \hat{\psi}_t\left(\theta\,(1+\theta) - \frac{2}{\beta}\right) + \tilde{y}_t\left(\frac{-2\,(1-\theta)\,(1-\theta\beta)}{\theta\beta}\zeta\right) \quad (23)\end{aligned}$$

7.6 Final Equations

If we collect the interest rate rule derived from the optimal policy problem in the indexation model, equations 20, 22 and 23 from the Kumhof et al model, together with the IS curve, we have the equations that describe the "comprehensive" model under analysis.

$$\begin{aligned}x_t &= E_t\left[x_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \varepsilon_t^D \\ i_t &= \left(\rho^{PC} + \gamma + \frac{\kappa\left(1-\rho^{PC}\right)\sigma}{\lambda}\right)\pi_t - \left(\rho^{PC}\gamma + \frac{\gamma\kappa\left(1-\rho^{PC}\right)\sigma}{\lambda}\right)\pi_{t-1} + \sigma\varepsilon_t^D + \varepsilon_t^{POL} \\ E_t\pi_{t+1} &= \pi_t\left(\frac{2}{\beta}-\theta\right) + \hat{v}_t\left((1-\theta)\,(1+\theta)\right) + \hat{\psi}_t\left(\theta\,(1+\theta) - \frac{2}{\beta}\right) + \tilde{y}_t\left(\frac{-2\,(1-\theta)\,(1-\theta\beta)}{\theta\beta}\zeta\right) \\ E_t\hat{v}_{t+1} &= \hat{v}_t + \left(\frac{1-\theta\beta}{\theta\beta}\right)^2\zeta\tilde{y}_t - \left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta}\pi_t + \left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta}\hat{\psi}_t \\ \hat{\psi}_t &= \theta\hat{\psi}_{t-1} + (1-\theta)\,\hat{v}_{t-1}\end{aligned}$$

$$x_t = E_t\left[x_{t+1}\right] - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \varepsilon_t^D$$

$$i_t = \theta_{i1}\pi_t + \theta_{i2}\pi_{t-1} + \sigma\varepsilon_t^D + \varepsilon_t^{POL}$$

$$E_t\pi_{t+1} = \theta_{\pi1}\pi_t + \theta_{\pi2}\hat{v}_t + \theta_{\pi3}\hat{\psi}_t + \theta_{\pi4}\tilde{y}_t$$

$$E_t\hat{v}_{t+1} = \theta_{v1}\hat{v}_t + \theta_{v2}\tilde{y}_t + \theta_{v3}\pi_t + \theta_{v4}\hat{\psi}_t$$

$$\hat{\psi}_t = \theta_{\psi1}\hat{\psi}_{t-1} + \theta_{\psi2}\hat{v}_{t-1}$$

or (gets a bit ugly)

$$\begin{aligned}
\frac{1}{\theta_{\pi1}}E_t\pi_{t+1} &= \pi_t + \frac{\theta_{\pi2}}{\theta_{\pi1}}\hat{v}_t + \frac{\theta_{\pi3}}{\theta_{\pi1}}\hat{\psi}_t + \frac{\theta_{\pi4}}{\theta_{\pi1}}\tilde{y}_t \\
&= \pi_t + \frac{\theta_{\pi2}}{\theta_{\pi1}}\hat{v}_t + \frac{\theta_{\pi3}}{\theta_{\pi1}}\hat{\psi}_t + \frac{\theta_{\pi4}}{\theta_{\pi1}}x_t + \frac{\theta_{\pi4}}{\theta_{\pi1}}\left(y_t^e - y_t^n\right) \\
&= \pi_t + \frac{\theta_{\pi2}}{\theta_{\pi1}}\hat{v}_t + \frac{\theta_{\pi3}}{\theta_{\pi1}}\hat{\psi}_t + \frac{\theta_{\pi4}}{\theta_{\pi1}}x_t + \frac{\theta_{\pi4}}{\kappa\theta_{\pi1}}\varepsilon_t^{PC} \\
\frac{1}{\theta_{v1}}E_t\hat{v}_{t+1} &= \hat{v}_t + \frac{\theta_{v2}}{\theta_{v1}}\tilde{y}_t + \frac{\theta_{v3}}{\theta_{v1}}\pi_t + \frac{\theta_{v4}}{\theta_{v1}}\hat{\psi}_t \\
&= \hat{v}_t + \frac{\theta_{v2}}{\theta_{v1}}x_t + \frac{\theta_{v3}}{\theta_{v1}}\pi_t + \frac{\theta_{v4}}{\theta_{v1}}\hat{\psi}_t + \frac{\theta_{v2}}{\theta_{v1}}\left(y_t^e - y_t^n\right) \\
&= \hat{v}_t + \frac{\theta_{v2}}{\theta_{v1}}x_t + \frac{\theta_{v3}}{\theta_{v1}}\pi_t + \frac{\theta_{v4}}{\theta_{v1}}\hat{\psi}_t + \frac{\theta_{v2}}{\kappa\theta_{v1}}\varepsilon_t^{PC}
\end{aligned}$$

or

$$\pi_t = \xi_{\pi1}E_t\pi_{t+1} + \xi_{\pi2}\hat{v}_t + \xi_{\pi3}\hat{\psi}_t + \xi_{\pi4}x_t + \xi_{\pi5}\varepsilon_t^{PC}$$

$$\hat{v}_t = \xi_{v1}E_t\hat{v}_{t+1} + \xi_{v2}x_t + \xi_{v3}\pi_t + \xi_{v4}\hat{\psi}_t + \xi_{v5}\varepsilon_t^{PC}$$

where

$$\begin{aligned}
\theta_{i1} &= \left(\rho^{PC} + \gamma + \frac{\kappa\left(1-\rho^{PC}\right)\sigma}{\lambda}\right) \quad \theta_{i2} = -\left(\rho^{PC}\gamma + \frac{\gamma\kappa\left(1-\rho^{PC}\right)\sigma}{\lambda}\right) \\
\theta_{\pi1} &= \frac{2}{\beta} - \theta \quad \theta_{\pi2} = (1-\theta)(1+\theta) \quad \theta_{\pi3} = \theta(1+\theta) - \frac{2}{\beta} \quad \theta_{\pi4} = \frac{-2(1-\theta)(1-\theta\beta)}{\theta\beta}\zeta \\
\theta_{v1} &= 1 \quad \theta_{v2} = \left(\frac{1-\theta\beta}{\theta\beta}\right)^2\zeta \quad \theta_{v3} = -\left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta} \quad \theta_{v4} = \left(\frac{1-\theta\beta}{\theta\beta}\right)^2\frac{\theta}{1-\theta}
\end{aligned}$$

8 C: Policy Rule Representations

The central bank's policy problem is

$$
\begin{aligned}
W &= \min_{\{i_{t+k}\}_{k=0:\infty}} E_t^{CB} \sum_{k=0}^{\infty} \beta^k \left[\left(\pi_{t+k} - \gamma_t \pi_{t+k-1} \right)^2 + \lambda x_{t+k}^2 \right] \\
& \quad s.t. \\
\pi_t - \gamma_t \pi_{t-1} &= \kappa x_t + \beta E_t \left[\pi_{t+1} - \gamma_t \pi_t \right] + \varepsilon_t \\
x_t &= E_t \left[x_{t+1} \right] - \frac{1}{\sigma} \left(i_t - E_t \left[\pi_{t+1} \right] \right) \\
\varepsilon_t &= \rho \varepsilon_{t-1} + \upsilon_t
\end{aligned}
$$

which gives rise to a familiar marginal condition for optimality (with quasidifferenced equation appearing due to indexation):

$$
x_t = -\frac{\kappa}{\lambda} \left(\pi_t - \gamma \pi_{t-1} \right) \tag{24}
$$

Substituting this into the Phillips Curve, solving forward and imposing a suitable TVC we obtain

$$
\begin{aligned}
\pi_t - \gamma \pi_{t-1} &= \lambda q \varepsilon_t \qquad (25) \\
q &\equiv \frac{1}{\lambda \left(1 - \beta \rho \right) + \kappa^2}
\end{aligned}
$$

Using the last two equations in combination with the IS curve one can obtain various representations of the interest rate rule that, in this model, would implement/be consistent with optimal policy:

- Rule 1

$$
\begin{aligned}
i_t &= \theta_{i1} \pi_t + \theta_{i2} \pi_{t-1} \\
\theta_{i1} &\equiv \rho + \frac{\kappa \sigma}{\lambda} \left(1 - \rho \right) + \gamma \\
\theta_{i2} &\equiv -\rho \gamma - \frac{\gamma \kappa \sigma}{\lambda} \left(1 - \rho \right)
\end{aligned}
$$

- Rule 2 (from Rule 1 but substituting out π_t using (25))

$$
i_t = \left(\theta_{i1} \gamma + \theta_{i2} \right) \pi_{t-1} + \theta_{i1} \lambda q \varepsilon_t
$$

- Rule 3

$$
\begin{aligned}
i_t &= \theta_{i1}\pi_{t-1} + \theta_{i2}\varepsilon_t \\
\theta_{i1} &\equiv \gamma^2 \\
\theta_{i2} &\equiv q\sigma\kappa\left(1-\rho\right) + \lambda\left(\gamma+\rho\right)
\end{aligned}
$$

- Rule 4

$$
\begin{aligned}
i_t &= \theta_{i1}\pi_t + \theta_{i2}\pi_{t-1} \\
\theta_{i1} &\equiv \gamma + \frac{\sigma\kappa\left(1-\rho\right)+\lambda\rho}{\lambda} \\
\theta_{i2} &\equiv -\frac{\gamma}{\lambda}\left(\sigma\kappa\left(1-\rho\right)+\lambda\rho\right)
\end{aligned}
$$

- Rule 5 (from Rule 4 but substituting out π_t using (25))

$$
i_t = \left(\theta_{i1}\gamma + \theta_{i2}\right)\pi_{t-1} + \theta_{i1}\lambda q\varepsilon_t
$$

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PAPERS PRESENTED AT THE CONFERENCE, IN ORDER OF PRESENTATION:

Title	Author(s) (presenter(s) in bold)
The Volatility of the Tradeable and Nontradeable Sectors: Theory and Evidence	**Laura Povoledo (UWE)**
The Interest Rate — Exchange Rate Nexus: Exchange Rate Regimes and Policy Equilibria	**Tatiana Kirsanova (Exeter)** co-authored with Christoph Himmels (Exeter)
The 'Puzzles' Methodology: En Route to Indirect Inference?	**Patrick Minford (Cardiff and CEPR)** with joint with Vo Phuong Mai Le (Cardiff) and Michael Wickens (Cardiff, York and CEPR)
Inflation, Human Capital and Tobin's q	**Parantap Basu (Durham)** with joint with Max Gillman (Cardiff) and Joseph Pearlman (London Metropolitan)
Endogenous Persistence in an Estimated New Keynesian Model Under Imperfect Information	**Joe Pearlman (London Metropolitan)** with joint with Paul Levine (Surrey), George Perendia (London Metropolitan) and Bo Yang (Surrey)
Expectational coordination with long-lived agents	**Roger Guesnerie (College de France)**
Expectations, Deflation Traps and Macroeconomic Policy	**George Evans (Oregon and St Andrews)**
Shocking Stuff: Technology, Hours and Factor Augmentation in Business-Cycle Models	**Peter McAdam (ECB)**
Monetary and Fiscal Policy under Deep Habits	**Ioana Moldovan (Glasgow)** with joint with Campbell Leith (Glasgow) and Raffaele Rossi (Glasgow)
Output Persistence from Monetary Shocks with Staggered Prices or Wages under a Taylor Rule	**Neil Rankin (York)** co-authored with Sebastiano Daros (Warwick and Bank of England)
The Suspension of the Gold Standard as Sustainable Monetary Policy	**Elisa Newby (Cambridge)**
Dynamic Games with Time Inconsistency	**Nicola Dimitri (Siena)**
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